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幂律流体在内管做行星运动的环空中流动时的内管法向应力分布

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摘 要:给出了运动双极坐标系中幂律流体在内管做行星运动的环空中流动时的控制方程以及内管法向应力分布的计算公式。在采用有限差分法对控制方程进行数值求解的基础上,利用计算公式对内管法向应力分布进行了数值计算;绘制了内管法向应力分布曲线,并对其影响因素进行了分析。结果表明,内管自转和公转速度、环空偏心度是影响内管法向应力分布的主要因素,而压力梯度的影响很小。

关键词:幂律流体; 偏心环空; 内管; 行星运动; 法向应力分布

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Normal stress distribution on the inner cylinder forced by power law fluid flowing in annulus with inner cylinder executing a planetary motion

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Abstract: The governing equations and calculation formulas of normal stress distribution on the inner cylinder were given in bipolar coordinate system when power law fluid flows in annulus with the inner cylinder executing a planetary motion. The governing equations were worked out by finite difference method, and then the normal stress distribution on the inner cylinder was calculated. The curves of normal stress distribution on the inner cylinder were plotted, and the affecting factors were analyzed. The results indicate that the rotation and revolution velocities of the inner cylinder, the eccentricity of the annulus are the main factors to influence the normal stress distribution on the inner cylinder, while the pressure gradient has less effect.

Key words: power law fluid; eccentric annulus; inner cylinder; planetary motion; normal stress distribution

在螺杆泵采油工况下,由于抽油杆自重以及偏心的影响,抽油杆不仅绕自身轴线自转,还绕油管的轴线公转,采出液在抽油杆和油管所形成的偏心环空中的流动可视为幂律流体在内管做行星运动的环空中的流动。Kazakia 等[1]给出了牛顿流体在内管做行星运动的环空中流动的解析解。季海军[2]通过实验验证了其建立的幂律流体在内管做行星运动的环空中流动的控制方程和数值求解方法的正确性。崔海清等[3]在先前的工作基础[4-6]上,对流体在内管做行星运动的环空中流动的二次流进行了计算与分析。笔者在以上前人研究的基础上,计算和分析幂律流体在内管做行星运动的环空中流动时的内管法向应力分布。

1 假设条件

(1)不可压缩幂律流体在无限长偏心环空中做等温层流流动。

(2)环空内、外管半径分别为 R_i 和 R_o,其轴线相互平行且间距为 e_p。外管静止,内管绕其轴线以等角速度 Ω_i 自转,同时又绕外管的轴线以等角速度 Ω 公转,即内管绕外管轴线做行星运动,如图 1 所示。

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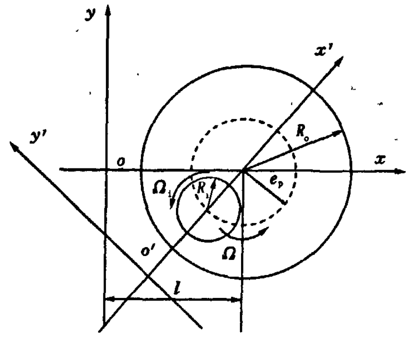


图 1　流体在内管做行星运动的环空中流动

(3) 作用于流体上的压力梯度为 G_{Dp}，且平行于环空内、外管轴线。

(4) 忽略惯性力的影响。

2　控制方程

崔海清等[7]通过公式推导，证明了用流函数和轴向速度表示的幂律流体在内管做行星运动的环空中流动的运动方程在运动直角坐标系与绝对直角坐标系中的形式一致。因此，运动双极坐标系(ξ,ζ,z)中幂律流体在内管做行星运动的环空中流动的运动方程[8-10]为

$$\begin{aligned}&(c^2+s^2)\eta\overline{\nabla}^4\psi+2\Big[\eta\frac{\partial(c^2+s^2)}{\partial\xi}+(c^2+s^2)\frac{\partial\eta}{\partial\xi}\Big]\times\\&\frac{\partial}{\partial\xi}(\overline{\nabla}^2\psi)+2\Big[\eta\frac{\partial(c^2+s^2)}{\partial\zeta}+(c^2+s^2)\frac{\partial\eta}{\partial\zeta}\Big]\times\\&\frac{\partial}{\partial\zeta}(\overline{\nabla}^2\psi)+\Big[\eta\overline{\nabla}^2(c^2+s^2)+2\frac{\partial(c^2+s^2)}{\partial\xi}\frac{\partial\eta}{\partial\xi}+\\&2\frac{\partial(c^2+s^2)}{\partial\zeta}\frac{\partial\eta}{\partial\zeta}\Big]\overline{\nabla}^2\psi+\Big[\frac{\partial(c^2+s^2)}{\partial\xi}\frac{\partial\eta}{\partial\xi}-\frac{\partial(c^2+s^2)}{\partial\zeta}\times\\&\frac{\partial\eta}{\partial\zeta}+(c^2+s^2)\Big(\frac{\partial^2\eta}{\partial\xi^2}-\frac{\partial^2\eta}{\partial\zeta^2}\Big)\Big]\Big(\frac{\partial^2\psi}{\partial\xi^2}-\frac{\partial^2\psi}{\partial\zeta^2}\Big)+\\&2\Big[\frac{\partial(c^2+s^2)}{\partial\zeta}\frac{\partial\eta}{\partial\xi}+\frac{\partial(c^2+s^2)}{\partial\xi}\frac{\partial\eta}{\partial\zeta}+2(c^2+s^2)\frac{\partial^2\eta}{\partial\xi\partial\zeta}\Big]\times\\&\frac{\partial^2\psi}{\partial\xi\partial\zeta}+\Big\{4\Big[\Big(\frac{\partial c}{\partial\xi}\Big)^2+\Big(\frac{\partial s}{\partial\xi}\Big)^2\Big]\frac{\partial\eta}{\partial\xi}+2\frac{\partial(c^2+s^2)}{\partial\zeta}\frac{\partial^2\eta}{\partial\xi\partial\zeta}+\\&\frac{\partial(c^2+s^2)}{\partial\xi}\Big(\frac{\partial^2\eta}{\partial\xi^2}-\frac{\partial^2\eta}{\partial\zeta^2}\Big)\Big\}\frac{\partial\psi}{\partial\xi}+\Big\{4\Big[\Big(\frac{\partial c}{\partial\zeta}\Big)^2+\Big(\frac{\partial s}{\partial\zeta}\Big)^2\Big]\times\\&\frac{\partial\eta}{\partial\zeta}+2\frac{\partial(c^2+s^2)}{\partial\xi}\frac{\partial^2\eta}{\partial\xi\partial\zeta}+\frac{\partial(c^2+s^2)}{\partial\zeta}\Big(\frac{\partial^2\eta}{\partial\zeta^2}-\frac{\partial^2\eta}{\partial\xi^2}\Big)\Big\}\frac{\partial\psi}{\partial\zeta}=0,\end{aligned}\tag{1}$$

$$\frac{\partial}{\partial\xi}\Big(\eta\frac{\partial w}{\partial\xi}\Big)+\frac{\partial}{\partial\zeta}\Big(\eta\frac{\partial w}{\partial\zeta}\Big)=-\frac{C^2}{c^2+s^2}G_{Dp}.\tag{2}$$

其中

$$C=\frac{[(R_i^2+R_o^2-e^2)^2-4R_i^2R_o^2]^{1/2}}{2e},$$

$$c=\operatorname{ch}\xi\cos\zeta-1,\ s=\operatorname{sh}\xi\sin\zeta,$$

$$\eta=k(I_2)^{n-1},$$

$$\begin{aligned}I_2=&\Big\{\frac{c^2+s^2}{C^2}\Big[\Big(\frac{\partial w}{\partial\xi}\Big)^2+\Big(\frac{\partial w}{\partial\zeta}\Big)^2\Big]+\\&\frac{1}{C^4}\Big[(c^2-s^2)\Big(\frac{\partial^2\psi}{\partial\zeta^2}-\frac{\partial^2\psi}{\partial\xi^2}\Big)-4cs\frac{\partial^2\psi}{\partial\xi\partial\zeta}-2\frac{\partial(cs)}{\partial\xi}\frac{\partial\psi}{\partial\zeta}-\\&2\frac{\partial(cs)}{\partial\zeta}\frac{\partial\psi}{\partial\xi}\Big]^2+\frac{4}{C^4}\Big[cs\Big(\frac{\partial^2\psi}{\partial\zeta^2}-\frac{\partial^2\psi}{\partial\xi^2}\Big)+(c^2-s^2)\frac{\partial^2\psi}{\partial\xi\partial\zeta}-\\&\frac{\partial(cs)}{\partial\xi}\frac{\partial\psi}{\partial\xi}+\frac{\partial(cs)}{\partial\zeta}\frac{\partial\psi}{\partial\zeta}\Big]^2\Big\}^{\frac{1}{2}}.\end{aligned}$$

式中，ψ 为流函数，m^2/s；w 为流体轴向速度，m/s；η 为粘度函数，Pa · s；k 为稠度系数，$Pa\cdot s^n$；n 为流性指数；I_2 为一阶 Rivlin-Ericksen 张量的第二不变量，s^{-1}。

根据流动的假设条件，流动边界条件可写为

$$\psi(\xi_i,0)=0,$$

$$\frac{\partial\psi}{\partial\xi}\Big|_{\xi=\xi_i}=-\frac{\Omega_iR_iC}{(c^2+s^2)^{\frac{1}{2}}},$$

$$\psi(\xi_o,\zeta)=\overline{\psi}=\text{const},$$

$$\frac{\partial\psi}{\partial\xi}\Big|_{\xi=\xi_o}=\frac{\Omega R_oC}{(c^2+s^2)^{\frac{1}{2}}},$$

$$w(\xi_i,\zeta)=0,\ w(\xi_o,\zeta)=0.$$

另外，为了求解方程(1)，(2)，还需要流体压力 p 的单值性条件

$$\oint_{\xi=\text{const}}\frac{\partial p}{\partial\zeta}\mathrm{d}\zeta=0.$$

其中

$$\begin{aligned}\frac{\partial p}{\partial\zeta}=&-\frac{\eta}{C^2}\Big[\frac{\partial(c^2+s^2)}{\partial\xi}\overline{\nabla}^2\psi+(c^2+s^2)\frac{\partial}{\partial\xi}\overline{\nabla}^2\psi\Big]+\\&\frac{2}{C^2(c^2+s^2)}\Big[2cs\frac{\partial\eta}{\partial\xi}-(c^2-s^2)\frac{\partial\eta}{\partial\zeta}\Big]\Big[-\frac{\partial(cs)}{\partial\xi}\frac{\partial\psi}{\partial\xi}+\\&\frac{\partial(cs)}{\partial\zeta}\frac{\partial\psi}{\partial\zeta}+(c^2-s^2)\frac{\partial^2\psi}{\partial\xi\partial\zeta}-cs\Big(\frac{\partial^2\psi}{\partial\xi^2}-\frac{\partial^2\psi}{\partial\zeta^2}\Big)\Big]-\\&\frac{1}{C^2(c^2+s^2)}\Big[(c^2-s^2)\frac{\partial\eta}{\partial\xi}+2cs\frac{\partial\eta}{\partial\zeta}\Big]\Big[2\frac{\partial(cs)}{\partial\zeta}\frac{\partial\psi}{\partial\xi}+\\&2\frac{\partial(cs)}{\partial\xi}\frac{\partial\psi}{\partial\zeta}+4cs\frac{\partial^2\psi}{\partial\xi\partial\zeta}+(c^2-s^2)\Big(\frac{\partial^2\psi}{\partial\xi^2}-\frac{\partial^2\psi}{\partial\zeta^2}\Big)\Big].\end{aligned}$$

其积分路线为环空内含内管的(ξ,ζ)平面上的封闭曲线 ξ = const。

3　内管法向应力分布

令 n_1,n_2 为垂直于环空内管的单位外法线向量 $\boldsymbol{n}$ 在 x,y 方向的方向余弦，则

$$n_1=\frac{c}{(c^2+s^2)^{\frac{1}{2}}},\quad n_2=-\frac{s}{(c^2+s^2)^{\frac{1}{2}}}.$$

令 x, y 方向的应力分量分别为 t_{n_1}, t_{n_2},则

$$t_{n_1} = \Pi_{11} n_1 + \Pi_{21} n_2, \tag{3}$$

$$t_{n_2} = \Pi_{12} n_1 + \Pi_{22} n_2. \tag{4}$$

其中

$$\Pi_{11} = -p + \tau_{xx},$$

$$\Pi_{12} = \tau_{xy} = \Pi_{21} = \tau_{yx},$$

$$\Pi_{22} = -p + \tau_{yy},$$

$$\tau_{xx} = 2\eta \frac{\partial^2 \psi}{\partial x \partial y},$$

$$\tau_{xy} = \tau_{yx} = \eta\left(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2}\right),$$

$$\tau_{yy} = -2\eta \frac{\partial^2 \psi}{\partial x \partial y}.$$

式中,Π_{ij} 为应力张量 $\boldsymbol{\Pi}$ 的物理分量,Pa;τ_{ij} 为偏应力张量 $\boldsymbol{T}$ 的物理分量,Pa。

将 $\tau_{xx}, \tau_{xy}, \tau_{yy}$ 的表达式转换到双极坐标系中为

$$\tau_{xx} = 2\eta \frac{1}{C^2}\left[-\frac{\partial(sc)}{\partial \xi}\frac{\partial \psi}{\partial \xi} + \frac{\partial(sc)}{\partial \zeta}\frac{\partial \psi}{\partial \zeta} + (c^2 - s^2) \times \frac{\partial^2 \psi}{\partial \xi \partial \zeta} + sc\left(\frac{\partial^2 \psi}{\partial \zeta^2} - \frac{\partial^2 \psi}{\partial \xi^2}\right)\right], \tag{5}$$

$$\tau_{xy} = \tau_{yx} = \eta \frac{1}{C^2}\left[-2\frac{\partial(sc)}{\partial \zeta}\frac{\partial \psi}{\partial \xi} - 2\frac{\partial(sc)}{\partial \xi}\frac{\partial \psi}{\partial \zeta} - 4sc\frac{\partial^2 \psi}{\partial \xi \partial \zeta} + (s^2 - c^2)\left(\frac{\partial^2 \psi}{\partial \xi^2} - \frac{\partial^2 \psi}{\partial \zeta^2}\right)\right], \tag{6}$$

$$\tau_{yy} = -\tau_{xx} = -2\eta \frac{1}{C^2}\left[-\frac{\partial(sc)}{\partial \xi}\frac{\partial \psi}{\partial \xi} + \frac{\partial(sc)}{\partial \zeta}\frac{\partial \psi}{\partial \zeta} + (c^2 - s^2)\frac{\partial^2 \psi}{\partial \xi \partial \zeta} + sc\left(\frac{\partial^2 \psi}{\partial \zeta^2} - \frac{\partial^2 \psi}{\partial \xi^2}\right)\right]. \tag{7}$$

将式(5) ~ (7) 代入式(3),(4) 中,求得幂律流体在内管做行星运动的环空中流动时流体作用在内管外壁上的内管法向应力为

$$T_i = t_{n_1} n_1 + t_{n_2} n_2 = \left\{-p + \frac{\eta}{C^2}\left[\frac{\partial(c^2 + s^2)}{\partial \zeta}\frac{\partial \psi}{\partial \xi} + \frac{\partial(c^2 + s^2)}{\partial \xi}\frac{\partial \psi}{\partial \zeta} + 2(c^2 + s^2)\frac{\partial^2 \psi}{\partial \xi \partial \zeta}\right]\right\}\Bigg|_{\xi = \xi_i}.$$

其中

$$p(\xi_i, \zeta) = \int_0^\zeta \frac{\partial p}{\partial \zeta}\bigg|_{\xi = \xi_i} d\zeta + p_0.$$

式中,p_0 为积分常数。

对内管 $\zeta = \zeta$ 处的法向应力 T_i 与 $\zeta = 0$ 处的法向应力 T_{i0} 作差消去常数 p_0[11],得到内管法向应力差 $T_i - T_{i0}$ 的计算公式为

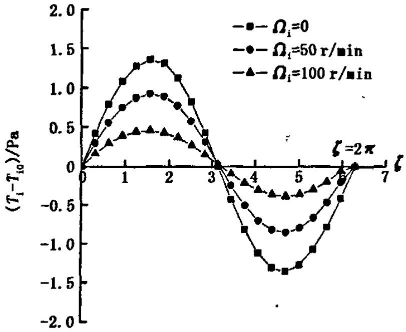
$$T_i - T_{i0} = -\int_0^\zeta \frac{\partial p}{\partial \zeta}\bigg|_{\xi = \xi_i} d\zeta + \frac{\eta}{C^2}\left[\frac{\partial(c^2 + s^2)}{\partial \zeta}\frac{\partial \psi}{\partial \xi} + \frac{\partial(c^2 + s^2)}{\partial \xi}\frac{\partial \psi}{\partial \zeta} + 2(c^2 + s^2)\frac{\partial^2 \psi}{\partial \xi \partial \zeta}\right]\bigg|_{\xi = \xi_i, \zeta = \zeta} - \frac{\eta}{C^2}\left[\frac{\partial(c^2 + s^2)}{\partial \zeta}\frac{\partial \psi}{\partial \xi} + \frac{\partial(c^2 + s^2)}{\partial \xi}\frac{\partial \psi}{\partial \zeta} + 2(c^2 + s^2)\frac{\partial^2 \psi}{\partial \xi \partial \zeta}\right]\bigg|_{\xi = \xi_i, \zeta = 0}. \tag{8}$$

4 算例分析

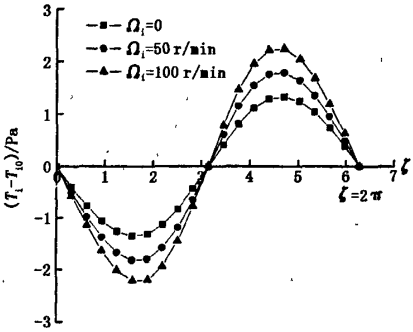
已知环空外管半径 $R_o = 0.024$ m,内管半径 $R_i = 0.0135$ m;流体密度 $\rho = 995$ kg/m^3,稠度系数 $k = 0.07385$ Pa · s$^{0.8332}$,流性指数 $n = 0.8332$。

首先,利用有限差分法对方程(1),(2) 进行求解,求出幂律流体在内管做行星运动的环空中流动的流函数分布[2,10];再利用数值积分方法对式(8) 进行数值计算,即可得到幂律流体在内管做行星运动的环空中流动时的内管法向应力分布。

分别变化内管自转角速度 Ω_i 和公转角速度 Ω(规定逆时针旋转方向为正)、环空偏心度 e_p($e_p = e/(R_o - R_i)$) 以及压力梯度 G_{Dp},分析其对内管法向应力分布的影响,结果如图 2 ~ 5 所示。



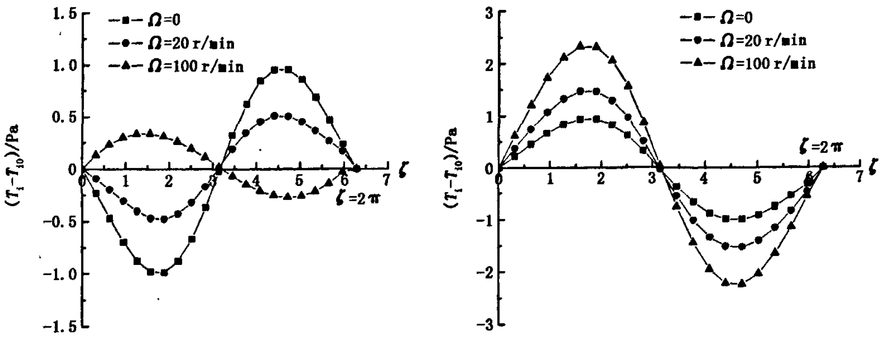
(a) Ω=50 r/min



(b) Ω=-50 r/min

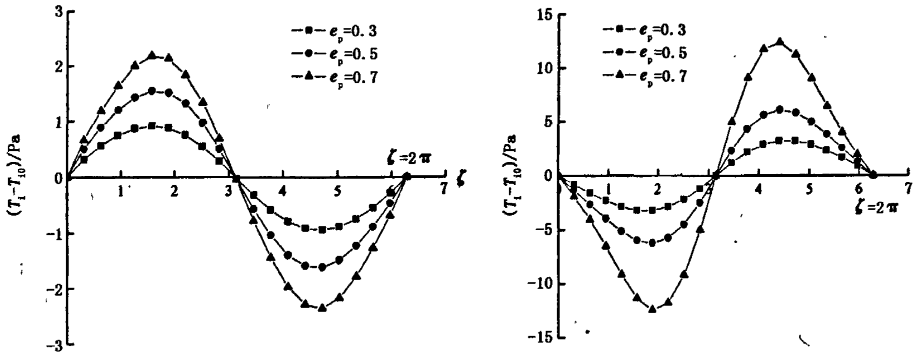
图 2　内管自转角速度 Ω_i 不同时的内管法向应力分布(G_{Dp} = 1.5 kPa/m, e_p = 0.2)

从图2中可以看出,当内管逆时针公转时,内管法向应力差 $T_i - T_{i0}$ 的绝对值随着内管自转角速度 Ω_i 的增大而减小;当内管顺时针公转时,内管法向应力差 $T_i - T_{i0}$ 的绝对值随着内管自转角速度 Ω_i 的增大而增大。



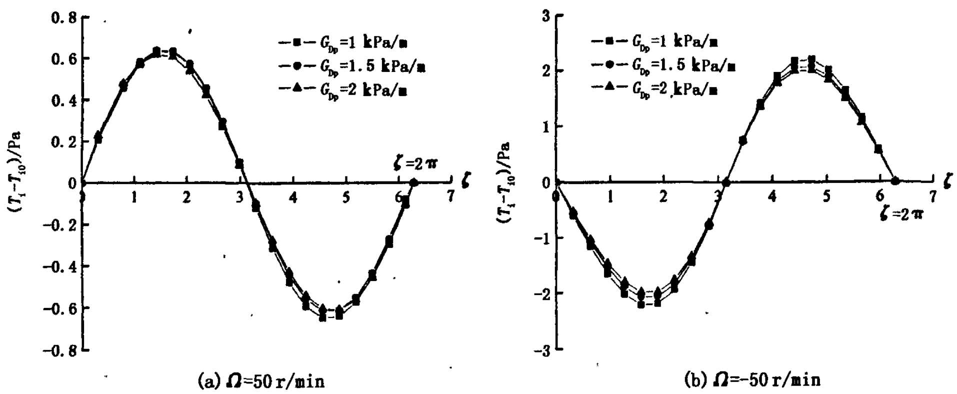
(a) Ω_i=117 r/min　(b) Ω_i=-117 r/min

图3　内管公转角速度 Ω 不同时的内管法向应力分布(G_{Dp} = 1.5 kPa/m, e_p = 0.2)



(a) Ω=50 r/min　(b) Ω=-50 r/min

图4　环空偏心度 e_p 不同时的内管法向应力分布(Ω_i = 80 r/min, G_{Dp} = 1.5 kPa/m)



(a) Ω=50 r/min　(b) Ω=-50 r/min

图5　压力梯度 G_{Dp} 不同时的内管法向应力分布(Ω_i = 80 r/min, e_p = 0.2)

从图3可以看出,当内管逆时针自转时,随着内管公转角速度 Ω 的增大,内管法向应力差 $T_i - T_{i0}$ 的绝对值先减小后增大;当内管顺时针自转时,内管法向应力差 $T_i - T_{i0}$ 的绝对值随着内管公转角速度 Ω 的增大而增大。

从图4,5可以看出,无论内管公转方向如何,内

管法向应力差 $T_i - T_{i0}$ 的绝对值均随环空偏心度 e_p 的增大而增大,随环空压力梯度 G_{Dp} 的增大而略有减小。

从图2～5可以看出,内管法向应力呈反对称分布。由此可知,内管上的法向应力是导致内管与外管之间偏磨的主要因素。

5 结束语

给出了运动双极坐标系中幂律流体在内管做行星运动的环空中流动时的控制方程和内管法向应力分布的计算公式。内管自转和公转角速度、环空偏心度是影响内管法向应力分布的主要因素,而压力梯度对其影响很小。分析结果对分析和防治螺杆泵采油井中抽油杆的偏磨具有指导意义。

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